

Atmiya institute of technology and Science for Diploma Studies
SEMISTER - II
Question Bank-1

Subject code:- 3320002/03
Subject Name:- Advance Mathematics(Group-I/II)
Branch:- Civil and Computer
Chap.2:- Function and Limit
Date:-31/01/2015

Last Date:-15/02/2015

Section : 1 Questions for mark 1

1. If $f(x) = \log(e^x)$ then $f(-1) = \dots\dots$
2. If $f(x) = \log(e^x)$ then $f(0) = \dots\dots$
3. $(f^{-1} \circ f)(x) = \dots\dots = (f \circ f^{-1})(x)$
4. If $f(x) = \frac{1-x}{1+x}$, then $f(x) \cdot f(-x) = \dots\dots$
5. If $f(x) = \frac{6x+5}{7}$, then $f^{-1}(x) = \dots\dots$
6. If $f(x) = \cos x$ then $f\left(\frac{3\pi}{2} - x\right) = \dots\dots$
7. If $f(x) = x^2$, then $f(x+1) - f(x-1) = \dots\dots$
8. If $f(x) = ax + \frac{1}{x}$ and $f\left(\frac{1}{5}\right) = \frac{28}{5}$, then $a = \dots\dots$
9. If $f(x) = \frac{x}{x-1}$, ($x \neq 1$), then $\frac{f(a)}{f(a+1)} = \dots\dots$
10. If $f(x) = \frac{x}{x+1}$, ($x \neq -1$), then $\frac{f\left(\frac{a}{b}\right)}{f\left(\frac{b}{a}\right)} = \dots\dots$
11. Find $f^{-1}(x)$ of following functions:
 - (a) $f(x) = 2x + 5$
 - (b) $f(x) = 7x + 2$
 - (c) $f(x) = \frac{6x+5}{7}$
 - (d) $f(x) = x^3$
 - (e) $f(x) = \sqrt{x^3}$
 - (f) $f(x) = \frac{1-x}{1+x}$
12. $\lim_{x \rightarrow 1} [x]^{\frac{1}{x-1}} = \dots\dots$
13. Find $\lim_{n \rightarrow \infty} \frac{\sin n\theta}{n} = \dots\dots$

14. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \dots$
15. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \dots$
16. $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \dots$
17. $\lim_{x \rightarrow 0} \frac{6^x - 2^x}{x} = \dots$
18. $\lim_{x \rightarrow \infty} x (\sqrt{x} - 1) = \dots$
19. $\lim_{x \rightarrow \pi} \frac{\tan x}{\pi - x} = \dots$
20. $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} = \dots$
21. $\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} = \dots, (a > 0)$.

Section : 2 Questions for mark 3

1. If $f(x) = 2x+1$ and $g(x) = x^2-2$ then find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $(\frac{f}{g})(x)$, $(\frac{g}{f})(x)$, $f^{-1}(x)$, $g^{-1}(x)$, $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f^{-1})(x)$, $(g \circ f^{-1})(x)$.
2. If $f(x) = \log\left(\frac{x}{x-1}\right)$, then show that $f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right)$.
3. If $f(x) = \frac{a+bx}{b+ax}$, then prove that $f(x) \cdot f\left(\frac{1}{x}\right) = 1$.
4. If $f(x) = \log_2 x$ and $g(x) = x^8$ then prove that $(f \circ g)(x) = 8f(x)$.
5. If $f(x) = \log\left(\frac{1-x}{1+x}\right)$, then prove that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.
6. Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$.
7. Evaluate: $\lim_{x \rightarrow 2} \frac{x^4 - 8x^2 + 16}{x^3 - 3x^2 + 4}$.
8. Find $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$.
9. $\lim_{x \rightarrow 0} \frac{3\sin x - \sin 3x}{x^3}$.
10. $\lim_{x \rightarrow 0} \frac{4^x - 3^x}{x}$.
11. $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x+2}\right)^x$.

Section : 3 Questions for mark 4

1. If $f(x) = \log\left(\frac{1-x}{1+x}\right)$, then prove that

(a) $f(x) = f^{-1}(x)$

(b) $f(x) \cdot f(-x) = 1$

(c) $f(x) + f\left(\frac{1}{x}\right) = 0$

(d) $f(x) - f\left(\frac{1}{x}\right) = 2f(x)$

(e) $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.

2. If $f(x) = \frac{x+3}{4x-5}$ and $t = \frac{3+5x}{4x-1}$ then show that $x = f(t)$.

3. If $f(x) = \frac{1+x}{1-x}$ then prove that $x(f(x)) + 1 = 0$.

4. Evaluate: $\lim_{x \rightarrow 2} \frac{x\sqrt{x} - 2\sqrt{2}}{x-2}$.

5. Evaluate: $\lim_{\theta \rightarrow \frac{\pi}{4}} \frac{\sin\theta - \cos\theta}{\theta - \frac{\pi}{4}}$.

6. Evaluate: $\lim_{x \rightarrow 0} \frac{x \log(1+x)}{1-\cos x}$.

7. Evaluate: $\lim_{n \rightarrow \infty} \frac{5^{n+1} - 7^{n+1}}{5^n + 7^n}$.

8. Prove that $\lim_{h \rightarrow 0} \frac{(5+2h)^{-1} - 5^{-1}}{h} = \frac{-2}{25}$.

9. Find $\lim_{x \rightarrow 0} \frac{x(1 - \sqrt{1-x^2})}{\sqrt{1-x^2}(\sin^{-1}x)^3}$.

10. Give the example of $\lim_{x \rightarrow a} (f(x) + g(x))$ is exists but $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ are not exists.

11. $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin\left(x - \frac{\pi}{3}\right)}{2\cos x - 1}$.

12. $\lim_{x \rightarrow 0} \frac{2x - 3\sin^{-1}x}{3x - 5\tan^{-1}x}$.