Atmiya institute of technology and Science for Diploma Studies
SEMISTER - II
Question Bank-1

Subject code:- 3320002/03
Subject Name:- Advance Mathematics(Group-I/II)
Branch:- Civil and Computer
Chap.3:- Differentiation and its Applications
Date:-17/03/2015 **Last Date:-25/04/2015**

Section : 1 Questions for mark 1

1. $\dfrac{d}{dx}x^n =$......
2. $\dfrac{d}{dx}(e^x + x^6 + e^e) =$......
3. $\dfrac{d}{dx}a^a =$......
4. $\dfrac{d}{dx}e^{5x} =$......
5. $\dfrac{d}{dx}x^x =$......
6. $\dfrac{d}{dx}\sqrt{x \sin x} =$......
7. $\dfrac{d}{dx}cotx =$......
8. $\left(\dfrac{d}{dx}sec^{-1}x\right)_{x=-3} =$......
9. $\dfrac{d}{dx}\tan^n x =$......
10. $\dfrac{d}{dx}(sin^2x + cos^2x) =$.....
11. $\dfrac{d}{dx}(3sinx - 4sin^3x) =$.....

12. $\frac{d}{dx}(sin^{-1}x + cos^{-1}x) =$.....

13. $\frac{d}{dx}\log(\cot x) =$......

14. $\frac{d}{dx}x^x =$.....

15. $\frac{d}{dx}e^{-\log x} =$......

16. $\frac{d}{dx}(x\log x) =$......

17. $\frac{d^2}{dx^2}(x\log x) =$......

18. $\frac{d}{dx}\log\sqrt{x^2+1} =$......

19. $f(x) = \log\sqrt{x^2+1}$, then $f'(0) =$......

20. If $x = at$ and $y = \frac{a}{t}$, then $\frac{dy}{dx} =$......

21. If $f(x) = e^{2x}$, then $f'(0) =$......

22. If $x^2 + y^2 = 29$, then $\frac{dy}{dx}$ at point $(2, 5)$=......

Section : 2 Questions for mark 3

1. Find $f'(x)$ for followings using first principal(By definition):
   (a) $f(x) = \cos x$.
   (b) $f(x) = \tan x$.
   (c) $f(x) = a^x$.
   (d) $f(x) = e^x$.
   (e) $f(x) = \log x$.
   (f) $f(x) = c$.

(g) $f(x) = x^3 + 5x$.

(h) $f(x) = x^2 + 2x - 1$.

(i) $f(x) = \sin^2 x$.

(j) $f(x) = \cos^4 x$.

(k) $f(x) = \tan^3 x$.

(l) $f(x) = f(x) = \dfrac{1 - \cos x}{\sin x}$.

(m) $f(x) = x \cos x$.

2. Find $\dfrac{dy}{dx}$ for followings:

(a) $y = \dfrac{4x^2 - 5x + 1}{x^5 - x^4}$.

(b) $y = \dfrac{x^2 - 1}{x^2 + 1}$.

(c) $y = e^x \sec x$.

(d) $y = e^3 x \cos 2x$.

(e) $y = \dfrac{\log x}{x}$.

(f) $y = \log\left(x + \sqrt{1 + x^2}\right)$.

(g) $\log y = x^x \log x$.

3. The equation of motion of particle is $s = t^3 - 6t^2 + 8t - 4$. Then find the velocity and acceleration when $t = 3sec$.

4. If $f(x) = \dfrac{\sin x}{\sin x - \cos x}$ and $g(x) = \dfrac{\cos x}{\sin x - \cos x}$, then show that its derivatives are equal.

5. If $x^3 + y^3 = x^3 y^3$, then prove that $\dfrac{dy}{dx} + \dfrac{y^4}{x^4} = 0$. Also prove that $\dfrac{dy}{dx} - \dfrac{x^2(1 - y^3)}{y^2(x^3) - 1} = 0$.

6. If $x^2 + xy + y^2 = 0$, then find $\dfrac{dy}{dx}$.

7. If $y = A\cos pt + B\sin pt$, then prove that $\dfrac{d^2y}{dt^2} + p^2y = 0$.

8. If $x = a\cos^4\theta$ and $y = n\sin^4\theta$, then prove that $\dfrac{dy}{dx} + \sqrt{\dfrac{by}{ax}} = 0$.

9. If $y = \log\left(\dfrac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}\right)$, then prove that $\sqrt{x^2+a^2}\dfrac{dy}{dx} = 2$.

Section : 3 Questions for mark 4

1. Find $\dfrac{dy}{dx}$ for followings:

   (a) $x + y = \sin(xy)$.

   (b) If $x = \dfrac{1}{2}\left(t + \dfrac{1}{t}\right)$ and $y = \dfrac{1}{2}\left(t - \dfrac{1}{t}\right)$.

   (c) $x - y = \sin(x+y)$.

   (d) $y = (\sin x)^x$.

   (e) $y = x^x \log x$.

   (f) $y = \cos x^x + \sin x^x$.

   (g) $y = (\log x)^{\cos x}$.

   (h) $x = a(\theta + \sin\theta)$ and $y = b(1 - \cos\theta)$.

   (i) $x = a\left(\cos t + \log\tan\dfrac{t}{2}\right)$ and $y = a\sin t$.

   (j) $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.

   (k) $y = \sin^{-1}\left(3x - 4x^2\right)$, $0 < x < \dfrac{1}{2}$.

   (l) $y = \tan^{-1}\dfrac{2x}{1-x^2}$, $x \neq \pm 1$.

   (m) $y = \cos^{-1}\dfrac{1-x^2}{1+x^2}$.

   (n) $y = \sin^{-1}\dfrac{2x}{1+x^2}$.

   (o) $y = \tan^{-1}\dfrac{3x - x^3}{1 - 3x^2}$, $|x| > \dfrac{1}{\sqrt{3}}$.

(p) $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right),\ \frac{1}{\sqrt{2}} < x < 1.$

2. $x = \sqrt{a^{\sin^{-1} t}}$ and $y = \sqrt{a^{\cos^{-1} t}}$, then prove that $\frac{dy}{dx} = \frac{-y}{x}$, when $|t| < 1$.

3. If $x = at^2$ and $y = 2at$, then prove that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$.

4. If $x^y y^x = 1$, then find $\frac{dy}{dx}$.

5. If $y = \frac{1}{x^2 - 5x + 6}$, then prove that $\frac{d^y}{dx^2} = \frac{2}{(x-3)^3} - \frac{2}{(x-2)^3}$.

6. If $y = e^{\tan -1x}$, then prove that $(1 + x^2)y_2 + (2x - 1)y_1 = 0$.

7. If $\log\left(x + \sqrt{1 + x^2}\right)$, then prove that $(1 + x^2)y_2 + xy_1 = 0$.

8. If $y = 2e^{3x} + 3e^{-2x}$, then prove that $y_2 - y_1 - 6y = 0$.

9. If $y = ae^{kx} + be^{-kx}$, then prove that $y_2 = k^2 y$.

10. If $y = a\cos(\log x) + b\sin(\log x)$, then prove that $x^2y_2 + xy_1 + y = 01$.

11. If $y = \sin(\sin x)$, then prove that $y_2 + y_1 \tan x + y\cos^2 x = 0$.

12. If $y = e^x \sin x$, then prove that $y_2 - 2y_1 + 2y = 0$.

13. If $y = \log\sin x$, then prove that $y_2 + y_1^2 + 1 = 0$.

14. If $y = e^{m\tan -1x}$, then prove that $(1 + x^2)y_2 + (2x - m)y_1 = 0$.

15. The distance of moving particle is given by $s = t^3 - 3t^2 + 4t + 3$. Find the velocity and acceleration at $t = 2$.

16. The distance of a moving particle is given by $s = t^3 - 3t^2 + 4t + 3$. Find the velocity at $t = 0$. And find acceleration at velocity, $v = 0$.

17. Equation of motion of a particle is $t^3 - 5t^2 + 3t$. When particle comes to rest? Find acceleration at that time.

18. Find Minimum and Maximum Value of the function $f(x) = x^3 - 4x^2 + 5x + 7$.

19. Find the maximum and minimum values of $f(x) = 3x^3 - 4x^2x + 5$.

20. Find the maximum and minimum values of $f(x) = x + \dfrac{1}{x}$.

21. Find the maximum and minimum values of $f(x) = x \log_e x$.