

ATMIYA INSTITUTE OF TECHNOLOGY AND SCIENCE FOR DIPLOMA
STUDIES, RAJKOT
MID SEM EXAM 2015
SUBJECT: ADVANCE MATHEMATICS
SUBJECT CODE: 3320002/03

Date: 02-03-2015

Marks: 30

Time:- 1-hour

Que:1 Fill in the blanks: [05]

1. If $f(x) = \sin x$, then $f\left(\frac{\pi}{2} + x\right) = \dots$

2. If $f(x) = \log e^x$, then $f(-x) = \dots$

3. $\lim_{x \rightarrow 0} \frac{2^x - 1}{x} = \dots$

4. $\lim_{x \rightarrow 0} \frac{\sin 3x}{\tan 2x} = \dots$

5. $\lim_{x \rightarrow 0} \log(1 + x)^{\frac{1}{x}} = \dots$

Que:2 Attempt any five questions: [10]

1. If $f(x) = \frac{ax + b}{bx + a}$, then prove that $f(x) \cdot f\left(\frac{1}{x}\right) = 1$.

2. If $f(x) = e^x$, then prove that *i*). $f(x + y) = f(x) \cdot f(y)$, and *ii*). $f(x - y) = \frac{f(x)}{f(y)}$.

3. Simplify: $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 1}$.

4. Simplify: $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^2 - 4}$.

5. Simplify: $\lim_{x \rightarrow 0} \frac{2 \tan x - 3x}{2x - 3 \sin x}$.

6. Simplify: $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^n$.

7. Simplify: $\lim_{n \rightarrow \infty} \frac{n^2 + 2n + 1}{2n^2 + 3}$.

Que:3 Attempt any five questions: [15]

1. If $f(x) = \log\left(\frac{x}{x-1}\right)$, then prove that $f(x+1) + f(x) = \log\left(\frac{x+1}{x-1}\right)$.

2. If $f(x) = \tan x$, then prove that

i). $f(2x) = \frac{2f(x)}{1 - (f(x))^2}$, and

ii). $f(x + y) = \frac{f(x) + f(y)}{1 - f(x)f(y)}$.

3. Simplify: $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^3 - 1}$.

4. Simplify: $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$.

5. Simplify: $\lim_{x \rightarrow 0} \frac{6^x - 3^x}{x}$.

6. Find: $\lim_{x \rightarrow \infty} \left(\frac{x+2}{x+3}\right)^x$.

7. Find: $\lim_{x \rightarrow 0} \frac{\sqrt{1-x^2} - \sqrt{1+x^2}}{x^2}$.

***** BEST OF LUCK *****