Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
DIPLOMA ENGINEERING – SEMESTER –II • EXAMINATION – SUMMER- 2015

Subject Code: 3320002   **Date:30/05/2015**
Subject Name: Basic Mathematics
Time:10:30 AM TO 1:00 PM   **Total Marks: 70**
Instructions:

1. **Attempt ALL questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of SIMPLE CALCULATOR is permissible.** (Scientific/Higher Version not allowed)
5. **English version is authentic.**

Q.1   Fill in the blanks using appropriate choice from the given options.   **14**

   1   An argument of 1+i ……………….

     (a)$\frac{\pi}{4}$ (b)$\frac{3\pi}{4}$ (c)$\frac{5\pi}{4}$ (d)$\frac{7\pi}{4}$

   2 i^9=………….

     (a)i (b)-i(c)1(d)-1

   3   If $\bar{z}=\cos\theta + i\sin\theta$ then z+$\bar{z}$ =……..

     (a)2isinθ (b)2cosθ (c)cosθ $(d)i\sin\theta$

   4 $\frac{1-i}{1+i}$ =…………

     (a)1 (b) -1 (c) i (d) -i

   5   If f$(x) = \cos x$ then f$(\pi - x)$=…….

     (a)$\cos x$ $(b) - \cos x$ $(c)\sin x$ $(d)) - \sin x$

   6 $\lim_{x\to0}\frac{\sin 3x}{x}$ =…………..

     (a) 1 (b)3 (c)$\frac{1}{3}$*(d) does not exist*

   7 $\frac{d(\sec x)}{dx}$…………..

     (a)$\sec x\ \tan x$ $(b)\ \csc x\ \cot x$ $(c) - \csc x\ \cot x$ (d)$- \sec x\ \tan x$

8 $\dfrac{d(\log \sin x)}{dx}$ =............

(a)$\cot x$ (b)$\tan x$ (c)$-\cot x$ (d)$-\tan x$

9 If x=$\cos\theta$, y=$\sin\theta$ then $\dfrac{dy}{dx}$ =...........

(a)$\cot\theta$ (b)$\tan\theta$ (c)$-\cot\theta$ (d)$-\tan\theta$

10 If $x^2 + y^2 = 1$ then $\dfrac{dy}{dx}$ =..........

(a) $\dfrac{x}{y}$ (b)$\dfrac{-x}{y}$ (c)$\dfrac{y}{x}$ (d)$\dfrac{-y}{x}$

11 $\int \dfrac{1}{x^2}dx$=......+c

(a) $\dfrac{1}{x}$ (b)$\dfrac{-1}{x}$ (c)$\dfrac{-1}{3x^3}$ (d)$\dfrac{1}{3x^3}$

12 $\int_0^1 e^x\, dx$ =........

(a) e-1 (b)1-e (c) e (d)-e

13 Order of $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx}$ +6=0 is.......

(a)1 (b)2 (c)-5 (d)6

14 Integrating factor of $\dfrac{dy}{dx} + \dfrac{y}{x}$ = x is ....

(a) $\log x$ (b) $\log\dfrac{1}{x}$ (c)$\dfrac{1}{x}$ (d)x

Q.2 **(a)** Attempt any two 06

1. Reduce $1 + \sqrt{3}i$ to modulus $-$ amplitude form

2. If $\sqrt{\dfrac{1+i}{1-i}}$ =a+ib then find $a^2 + b^2$

3. If $\alpha + i\beta = \dfrac{1}{a+ib}$ then prove that $(\alpha^2 + \beta^2)(a^2 + b^2) = 1$

(b) Attempt any two 08

1 If $f(x) = \dfrac{1-x}{1+x}$ then prove that f(y) = x

2. Evaluate : $\lim_{x\to 1} \dfrac{x^3-x^2+x-1}{x^2-1}$

3. Evaluate : $\lim_{x \to 0} \frac{e^x + \sin x - 1}{x}$

Q.3 **(a)** Attempt any two   **06**

1. Differentiate
e^x $with$ $respect$ to x $using$ $first$ $principle$ of $differentiation$

2. Find $\frac{dy}{dx}$ if y=log(sec x + tan x)

3. Find $\frac{dy}{dx}$ aif $x = a(1 + \cos\theta)$ and y=b($\theta + sin\theta$)

(b) Attempt any two   **08**

1. If y=log $sinx$ $then$ $prove$ $that$ $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0$

2. Equation of motion of a particle in a straight line is S=$t^3 - 6t^2 + 9t$

Find velocity and acceleration at t=3 sec

3. Find the maximum and minimum values of values of $x^3 - 3x + 11$

Q.4 **(a)** Attempt any two   **06**

1. Evaluate :$\int \left(x + \frac{1}{x}\right)^2 dx$

2. Evaluate : $\int e^{sinx} \cos x$ dx

3. Evaluate : $\int x \sin x \, dx$

(b) Attempt any two   **08**

1. Evaluate :$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

2. Evaluate :$\int_0^1 \frac{\tan^{-1} x}{(1+x^2)}$ dx

3. Find the area of a region bounded by the curve y=x^2 , x-axis, x=1 and x=2

Q.5 **(a)** Attempt any two   **06**

1. If f(x) = tan x then prove that $f(A + B) = \frac{f(A) + f(B)}{1 - f(A)f(B)}$

2. Find the order and degree of $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho \left(\frac{d^2y}{dx^2}\right)^2$

3. Form the differential equation whose general solution is

   Y=$A \cos x + B \sin X$

(b) Attempt any two   08

1. Solve : $\frac{dy}{dx} = \frac{y}{x}$

2. Solve : $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

3. Solve : $\cos x \frac{dy}{dx} + y = \sin x$

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ગુજરાતી

પ્રશ્ન. ૧   યોગ્ય વિકલ્પ શોધી ખાલી જગ્યા પુરો   ૧૪

૧.   1+i નો કોણાંક..............

    (અ)$\frac{\pi}{4}$ (બ)$\frac{3\pi}{4}$ (ક)$\frac{5\pi}{4}$ (S)$\frac{7\pi}{4}$

૨. i^9=.............

    (અ) i (બ) -i(ક) 1(S) -1

3.   જો $\bar{z}$=$\cos \theta + i \sin \theta$ તો z+$\bar{z}$ =........

    (અ)2i$\sin \theta$ (બ)2$\cos \theta$ (ક)$\cos \theta$ (S)$i \sin \theta$

૪. $\frac{1-i}{1+i}$ =............

    (અ)1 (બ) -1 (ક) i (S) -i

પ.   જો f$(x) = \cos x$ તો f$(\pi - x)$=.......

    (અ)$\cos x$ (બ) $- \cos x$ (ક)$\sin x$ (S) $- \sin x$

૬. $\lim_{x \to 0} \frac{\sin 3x}{x} =$..............

(અ)1 (બ)3 (ક)$\frac{1}{3}$ (ડ) અસ્તિત્વ નથી

૭. $\frac{d(\sec x)}{dx}$..............

(અ)$\sec x \ \tan x$ (બ) $\csc x \ \cot x$ (ક) $-\csc x \cot x$ (ડ)$- \sec x \tan x$

૮. $\frac{d(\log \sin x)}{dx} =$............

(અ)$\cot x$ (બ) $\tan x$ (ક) $- \cot x$ (ડ) $- \tan x$

૯. જો x=$\cos \theta$, y=$\sin \theta$ તો $\frac{dy}{dx} =$..........

(અ)$\cot \theta$ (બ) $\tan \theta$ (ક) $- \cot \theta$ (ડ) $- \tan \theta$

૧૦ જો $x^2 + y^2 = 1$ તો $\frac{dy}{dx} =$..........

(અ)$\frac{x}{y}$ (બ) $\frac{-x}{y}$ (ક) $\frac{y}{x}$ (ડ) $\frac{-y}{x}$

૧૧ $\int \frac{1}{x^2}$dx=......+c

(અ)$\frac{1}{x}$ (બ) $\frac{-1}{x}$ (ક)$\frac{-1}{3x^3}$ (ડ) $\frac{1}{3x^3}$

૧૨ $\int_0^1 e^x \, dx =$........

(અ)e-1 (બ)1-e (ક) e (ડ)-e

૧૩ $\frac{d^2y}{dx^2} - 5\frac{dy}{dx}$ +6=0 ની કક્ષા ....

(અ)1 (b)2 (c)-5 (d)6

૧૪ $\frac{dy}{dx} + \frac{y}{x} =$x નો સંકલ્યકારકા અવયવ...........

(અ)$\log x$ (બ) $\log \frac{1}{x}$ (ક)$\frac{1}{x}$ (ડ)x

પ્રશ્ન. ૨ અ કોઇપણ બે ગણો 0૬

(૧) $1 + \sqrt{3}i$ નું માનાંક-કોણાંક-સ્વરૂપમાં રૂપાંતર કરો

(૨) જો $\sqrt{\frac{1+i}{1-i}}$ =a+ib તો $a^2 + b^2$ મેળવો

(3) જો $\alpha + i\beta = \frac{1}{a+ib}$ તો સાબિત કરો કે $(\alpha^2 + \beta^2)(a^2 + b^2) = 1$

બ કોઇપણ બે ગણો   ૦૮

(૧) જો $f(x) = \frac{1-x}{1+x}$ તો સાબિત કરો કે $f(y) = x$

(૨)મેળવો : $\lim_{x \to 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

(3) મેળવો : $\lim_{x \to 0} \frac{e^x + \sin x - 1}{x}$

પ્ર.3 અ કોઇપણ બે ગણો   ૦૬

(૧) e^x નું વિકલન x ની સાપેક્ષ વિકલનનાં પ્રથમ સિધ્ધાંત થી કરો

(૨)જો y=log(sec x + tan x) તો $\frac{dy}{dx}$ મેળવો

(3)જો $x = a(1 + \cos\theta)$ and y=$b(\theta + sin\theta)$ તો $\frac{dy}{dx}$ મેળવો

પ્ર.૪ અ કોઇપણ બે ગણો   ૦૬

(૧) મેળવો : $\int \left(x + \frac{1}{x}\right)^2 dx$

(૨) મેળવો : $\int e^{sinx} \cos x$ dx

(3)મેળવો : $\int x \sin x \, dx$

બ કોઇપણ બે ગણો   ૦૮

(૧) મેળવો : $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

(૨) મેળવો : $\int_0^1 \frac{\tan^{-1} x}{(1+x^2)} dx$

(3) y=x^2 , X- અક્ષ, x=1 અને x=2 થી ઘેરાયેલાં ક્ષેત્રનું ક્ષેત્રફળ મેળવો

પ્ર. ૫ અ   કોઇપણ બે ગણો   ૦૬

(૧) જો f(x) = $\tan x$ તો સાબિત કરો કે $f(A+B) = \frac{f(A)+f(B)}{1-f(A)f(B)}$

(૨) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \rho \left(\frac{d^2y}{dx^2}\right)^2$ ની કક્ષા અને પરિમાણ મેળવો.

(3)જેનો સામાન્ય ઉકેલ Y=$A \cos x + B \sin X$ તે વિકલ સમીકરણ મેળવો.

બ   કોઇપણ બે ગણો   ૦૮

(૧) ઉકેલો : $\frac{dy}{dx} = \frac{y}{x}$

(૨) ઉકેલો : $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

(3) ઉકેલો : $\cos x \frac{dy}{dx} + y = \sin x$