

GUJARAT TECHNOLOGICAL UNIVERSITY
Diploma Engineering – SEMESTER – II • Examination – WINTER • 2014

Subject Code: 3320002

Date: 22-12-2014

Subject Name: Advanced Mathematics (Group-1)

Time: 10:30 am - 01:00 pm

Total Marks: 70

Instructions:

1. Attempt ALL questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of SIMPLE CALCULATOR is permissible. (Scientific/Higher Version not allowed)
5. English version is authentic.

Q.1

Fill in the blanks using appropriate choice from the given options.

14

- 1 Amplitude of $1 - \sqrt{3}i$ is.....
 (a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{4\pi}{3}$
- 2 $[\cos \theta + i \sin \theta]^4 + [\cos \theta + i \sin \theta]^{-4} = \dots\dots\dots$
 (a) $2 \sin 4\theta$ (b) $2i \sin 4\theta$ (c) $2i \cos 4\theta$ (d) $2 \cos 4\theta$
- 3 $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \dots\dots\dots$
 (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0
- 4 $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \dots\dots\dots$
 (a) e (b) 1 (c) -1 (d) 0
- 5 $\frac{d(\cot x)}{dx} = \dots\dots\dots$
 (a) $\sec^2 x$ (b) $\cos ec^2 x$ (c) $-\sec^2 x$ (d) $-\cos ec^2 x$
- 6 $\frac{d(\log \sin x)}{dx} = \dots\dots\dots$
 (a) $\cos ecx$ (b) $-\cos ecx$ (c) $\cot x$ (d) $-\cot x$
- 7 $\frac{d\left(\frac{u}{v}\right)}{dx} = \dots\dots\dots$
 (a) $\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ (b) $\frac{v \frac{du}{dx} + u \frac{dv}{dx}}{v^2}$ (c) $\frac{u \frac{dv}{dx} - v \frac{du}{dx}}{v^2}$ (d) $\frac{u \frac{dv}{dx} + v \frac{du}{dx}}{v^2}$
- 8 $\frac{d(e^{-\log x})}{dx} = \dots\dots\dots$
 (a) $\frac{1}{x}$ (b) $-\frac{1}{x}$ (c) $-\frac{1}{x^2}$ (d) $\frac{1}{x^2}$
- 9 $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \dots\dots\dots + c$

(a) $\frac{1}{a} \cos^{-1}\left(\frac{x}{a}\right)$ (b) $\cos^{-1}\left(\frac{x}{a}\right)$ (c) $\frac{1}{a} \sin^{-1}\left(\frac{x}{a}\right)$ (d) $\sin^{-1}\left(\frac{x}{a}\right)$

10 $\int \frac{1}{x^2 + 25} dx = \dots\dots + c$

(a) $\tan^{-1}\left(\frac{x}{5}\right)$ (b) $\frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right)$ (c) $\frac{1}{5} \tan^{-1}\left(\frac{5}{x}\right)$ (d) $\tan^{-1}\left(\frac{5}{x}\right)$

11 $\int \cos(ax+b) dx \dots\dots + c$

(a) $\sin(ax+b)$ (b) $\frac{-\sin(ax+b)}{a}$ (c) $\frac{\sin(ax+b)}{b}$ (d) $\frac{\sin(ax+b)}{a}$

12 $\int_0^1 e^x dx = \dots\dots + c$

(a) e-1 (b) e+1 (c) 1-e (d) e

13 The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 3\left(\frac{dy}{dx}\right)^2 - 5y = 0$

are..... respectively.

(a) 2,3 (b) 3,2 (c) 1,2 (d) 2,1

14 Integrating factor of $\frac{dy}{dx} + \frac{2y}{x} = e^x$ is

(a) e^x (b) e^{x^2} (c) 2x (d) x^2

Q.2 (a) Attempt any two

06

1. If $f(x) = \frac{a+bx}{b+ax}$ then prove that $f(x) \cdot f\left(\frac{1}{x}\right) = 1$

2. Evaluate : $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$

3. Evaluate : $\lim_{x \rightarrow 0} \frac{4^x - 3^x}{x}$

(b) Attempt any two

08

1. If $f(x) = \frac{1-x}{1+x}$ then prove that

(i) $f(x) + f\left(\frac{1}{x}\right) = 0$ (ii) $f(x) - f\left(\frac{1}{x}\right) = 2f(x)$

2. Evaluate : $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3}$

3. Evaluate : $\lim_{x \rightarrow \infty} \left[\frac{x+1}{x+2} \right]^x$

Q.3 (a) Attempt any two

06

1. Differentiate $x^3 - x^2 + x - 1$ with respect to x using first principle of differentiation

2. Find $\frac{dy}{dx}$ if $y = \log(\cos ex - \cot x)$

3. Find $\frac{dy}{dx}$ if $y = \frac{3+4 \sin x}{4+3 \sin x}$

(b) Attempt any two

08

1. Find $\frac{dy}{dx}$ if $x = a \cos^2 \theta$, $y = b \sin^2 \theta$
2. If $y = 2e^{3x} + 3e^{-2x}$ then prove that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$
3. Equation of a motion of a particle is $s = 2t^3 - 3t^2 - 12t + 5$. Find the velocity and acceleration of a particle at $t=1$ sec and $t=2$ sec

Q.4 (a) Attempt any two **06**

1. Evaluate : $\int \frac{2+3\sin x}{\cos^2 x} dx$
2. Evaluate : $\int \sin^5 x \cos x dx$
3. Evaluate : $\int xe^x dx$

(b) Attempt any two **08**

1. Evaluate : $\int_1^e \frac{(\log x)^2}{x} dx$
2. Evaluate : $\int_0^{\frac{\pi}{2}} \log \tan x dx$
3. Find the area of a region bounded by $y = 3x^2$, $x=2$, $x=3$ and x -axis

Q.5 (a) Attempt any two **06**

1. Find the modulus and amplitude of $\frac{1+i}{1-i}$
2. Prove that $(1 + \cos \theta + i \sin \theta)^n = 2^n \cos^n \left(\frac{\theta}{2} \right) \left[\cos \frac{n\theta}{2} + i \sin \frac{n\theta}{2} \right]$
3. Solve : $(1+x^2)dx = (1+y^2)dy$

(b) Attempt any two **08**

1. Solve : $(x^2 + y^2)dx = 2xydy$
2. Solve : $\frac{dy}{dx} + \frac{2y}{x} = \sin x$
3. Find the cube roots of 1 using De Moivre's Theorem

(1) $1 - \sqrt{3}i$ નો કોણાંક

(અ) $\frac{\pi}{3}$ (બ) $-\frac{\pi}{3}$ (ક) $\frac{2\pi}{3}$ (ડ) $\frac{4\pi}{3}$

(2) $[\cos \theta + i \sin \theta]^4 + [\cos \theta + i \sin \theta]^{-4} = \dots\dots\dots$

(અ) $2 \sin 4\theta$ (બ) $2i \sin 4\theta$ (ક) $2i \cos 4\theta$ (ડ) $2 \cos 4\theta$

(3) $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \dots\dots\dots$

(અ) 1 (બ) 2 (ક) $\frac{1}{2}$ (ડ) 0

(4) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \dots\dots\dots$

(અ) e (બ) 1 (ક) -1 (ડ) 0

(5) $\frac{d(\cot x)}{dx} = \dots\dots\dots$

(અ) $\sec^2 x$ (બ) $\operatorname{cosec}^2 x$ (ક) $-\sec^2 x$ (ડ) $-\operatorname{cosec}^2 x$

(6) $\frac{d(\log \sin x)}{dx} = \dots\dots\dots$

(અ) $\operatorname{cosec} x$ (બ) $-\operatorname{cosec} x$ (ક) $\cot x$ (ડ) $-\cot x$

(7) $\frac{d\left(\frac{u}{v}\right)}{dx} = \dots\dots\dots$

(અ) $\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ (બ) $\frac{v \frac{du}{dx} + u \frac{dv}{dx}}{v^2}$ (ક) $\frac{u \frac{dv}{dx} - v \frac{du}{dx}}{v^2}$ (ડ) $\frac{u \frac{dv}{dx} + v \frac{du}{dx}}{v^2}$

(8) $\frac{d(e^{-\log x})}{dx} = \dots\dots\dots$

(અ) $\frac{1}{x}$ (બ) $-\frac{1}{x}$ (ક) $-\frac{1}{x^2}$ (ડ) $\frac{1}{x^2}$

(9) $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \dots\dots\dots + c$

(અ) $\frac{1}{a} \cos^{-1}\left(\frac{x}{a}\right)$ (બ) $\cos^{-1}\left(\frac{x}{a}\right)$ (ક) $\frac{1}{a} \sin^{-1}\left(\frac{x}{a}\right)$ (ડ) $\sin^{-1}\left(\frac{x}{a}\right)$

(10) $\int \frac{1}{x^2 + 25} dx = \dots\dots\dots + c$

(અ) $\tan^{-1}\left(\frac{x}{5}\right)$ (બ) $\frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right)$ (ક) $\frac{1}{5} \tan^{-1}\left(\frac{5}{x}\right)$ (ડ) $\tan^{-1}\left(\frac{5}{x}\right)$

(11) $\int \cos(ax + b) dx = \dots\dots\dots + c$

(અ) $\sin(ax + b)$ (બ) $\frac{-\sin(ax + b)}{a}$ (ક) $\frac{\sin(ax + b)}{b}$ (ડ) $\frac{\sin(ax + b)}{a}$

$$(12) \int_0^1 e^x dx = \dots\dots\dots + c$$

(અ) e-1 (બ) e+1 (ક) 1-e (ડ) e

(13) વિકલ સમીકરણ $\left(\frac{d^2y}{dx^2}\right)^3 + 3\left(\frac{dy}{dx}\right)^2 - 5y = 0$ ની કક્ષા અને પરિમાણ અનુક્રમેછે.

(અ) 2,3 (બ) 3,2 (ક) 1,2 (ડ) 2,1

(14.) $\frac{dy}{dx} + \frac{2y}{x} = e^x$ નો સંકલ્પ અવયવછે

(અ) e^x (બ) e^{x^2} (ક) $2x$ (ડ) x^2

પ્રશ્ન. ૨ અ કોઇપણ બે ગણો

6

(1) જો $f(x) = \frac{a+bx}{b+ax}$ તો સાબિત કરો કે $f(x) \cdot f\left(\frac{1}{x}\right) = 1$

(2) $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$ મેળવો

(3) $\lim_{x \rightarrow 0} \frac{4^x - 3^x}{x}$ મેળવો

બ કોઇપણ બે ગણો.

8

(1) જો $f(x) = \frac{1-x}{1+x}$ તો સાબિત કરો કે

(i) $f(x) + f\left(\frac{1}{x}\right) = 0$ (ii) $f(x) - f\left(\frac{1}{x}\right) = 2f(x)$

(2) $\lim_{x \rightarrow 0} \frac{3\sin x - \sin 3x}{x^3}$ મેળવો

(3) $\lim_{x \rightarrow \infty} \left[\frac{x+1}{x+2}\right]^x$ મેળવો

પ્રશ્ન. ૩ અ કોઇપણ બે ગણો

6

(1) $x^3 - x^2 + x - 1$ નું ની સાપેક્ષ વિકલન વિકલન ના પ્રથમ સિધ્ધાંત થી કરો

(2) જો $y = \log(\cos ex - \cot x)$ તો $\frac{dy}{dx}$ મેળવો

(3) જો $y = \frac{3+4\sin x}{4+3\sin x}$ તો $\frac{dy}{dx}$ મેળવો

બ કોઇપણ બે ગણો

8

(1) જો $x = a\cos^2 \theta$, $y = b\sin^2 \theta$ તો $\frac{dy}{dx}$ મેળવો

(2) જો $y = 2e^{3x} + 3e^{-2x}$ તો સાબિત કરો કે $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$

(3) કણની ગતિનું સુત્ર $s = 2t^3 - 3t^2 - 12t + 5$ છે. $t=1$ sec અને $t=2$ sec આગળ કણનો વેગ અને પ્રવેગ મેળવો

પ્રશ્ન. ૪ અ કોઈપણ બે ગણો 6

(1) $\int \frac{2+3\sin x}{\cos^2 x} dx$ મેળવો

(2) $\int \sin^5 x \cos x dx$ મેળવો

(3) $\int xe^x dx$ મેળવો

બ કોઈપણ બે ગણો 8

(1) $\int_1^e \frac{(\log x)^2}{x} dx$ મેળવો

(2) $\int_0^{\frac{\pi}{2}} \log \tan x dx$ મેળવો

(3) $y = 3x^2$, $x=2$, $x=3$ અને x -અક્ષ થી ઘેરાયેલાં પ્રદેશ નું ક્ષેત્રફળ મેળવો

પ્રશ્ન. ૫ અ કોઈપણ બે ગણો 6

(1) $\frac{1+i}{1-i}$ નો માનાંક અને કોણાંક મેળવો

(2) સાબિત કરો કે $(1 + \cos \theta + i \sin \theta)^n = 2^n \cos^n \left(\frac{\theta}{2} \right) \left[\cos \frac{n\theta}{2} + i \sin \frac{n\theta}{2} \right]$

(3) સમીકરણ $(1+x^2)dx = (1+y^2)dy$ ઉ કેલો

બ કોઈપણ બે ગણો 8

(1) સમીકરણ $(x^2 + y^2)dx = 2xydy$ ઉ કેલો

(2) સમીકરણ $\frac{dy}{dx} + \frac{2y}{x} = \sin x$ ઉ કેલો

(3) ડ-મોઇવર્સ પ્રમેય નો ઉપયોગ કરી 1 નું ઘનમુળ મેળવો.
