GUJARAT TECHNOLOGICAL UNIVERSITY

Diploma Engineering - SEMESTER–II • EXAMINATION – WINTER 2013

Subject Code: 3320002   **Date: 30-12-2013**

Subject Name: Advanced]Mathematics (Group-I)

Time: 10:30 am - 01:00 pm   **Total Marks: 70**

Instructions:
1. **Attempt any five questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of SIMPLE CALCULATOR is permissible.** (Scientific/Higher Version not allowed)
5. **English version is authentic.**

Q.1 Fill in the blanks using appropriate choice from the given options.   [14]

(1) Inverse of the number 3+4i is _____.

    (a) $\dfrac{3+4i}{5}$ (b) $\dfrac{3-4i}{25}$ (c) $\dfrac{3+4i}{25}$ (d) $\dfrac{4+3i}{25}$

(2) If $f(x) = x^3 - 1$ then $f(2) + f(-3) =$ _____

    (a) 35   (b) --21   (c) 28   (d) None of This

(3) $\lim\limits_{x \to 0} \left(\dfrac{\tan 5x}{\sin 3x} \right) =$ _____

    (a) $\dfrac{3}{5}$ (b) $\dfrac{5}{3}$ (c) 1   (d) None of This

(4) $\dfrac{d}{dx}(x \log x) =$ _____

    (a) $1 + \log x$ (b) $1 - \log x$ (c) $x + \log x$ (d) None of This

(5) $x^2 + y^2 = 29$ Then $\dfrac{dy}{dx}$ at the point (2, 5) = _____

    (a) $\dfrac{2}{5}$ (b) $-\dfrac{2}{5}$ (c) $-\dfrac{5}{2}$ (d) None of This

(6) $\int (\sin^2 x + \cos^2 x) dx =$ _____ + C.

    (a) x   (b) 2x   (c) tan x   (d) None of This

(7) $\int \log x\, dx =$ _____ +C

    (a) $x \log x + x$ (b) $x \log x - x$ (c) $\log |\cos \theta|$ (d) None of This

(8) $\int\limits_{-1}^{1} (x^2 + 1) dx =$ _____

    (a) 8/3   (b) 3/8   (c) 0   (d) None of This

(9) Order of the differential equation $\dfrac{d^2 y}{dx^2} + \left(\dfrac{dy}{dx} \right)^2 = xy$ is given by_____

    (a) 1   (b) 2   (c) 3   (d) None of This

(10) $Z = 5i - 3$ Then $\overline{Z} =$ _____

    (a) $3i - 5$ (b) $5i + 3$ (c) $-5i - 3$ (d) None of This

(11) $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{x - 2} =$ _____

    (a) 14   (b) 12   (c) 32   (d) None of This

(12) If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\dfrac{dy}{dx} =$ _____

    (a) $-\sqrt{\dfrac{y}{x}}$ (b) $-\sqrt{\dfrac{x}{y}}$ (c) $\sqrt{\dfrac{x}{y}}$ (d) None of This

(13) If $y = \sec\theta$ and $x = \tan\theta$ Then $\dfrac{d^2 y}{dx^2} =$ _____

    (a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) None of This

(14) The Integrating Factor of the equation $\dfrac{dy}{dx} + y = 3x$ is _____

    (a) 1   (b) 2   (c) e^x (d) log x

Q.2 (A) Do as directed (Any two)   [06]

    (1) If $f(x) = \log x$,then Prove That, (i) $f(xy) = f(x) + f(y)$ and

$$\text{(ii) } f\left(\frac{x}{y}\right) = f(x) - f(y)$$

    (2) Evaluate: $\lim\limits_{x \to 1}\left(\dfrac{x^2 - 4x + 3}{x^2 + 2x - 3}\right)$.

    (3) Calculate: $\lim\limits_{x \to 0}\left(\dfrac{e^x + \sin 2x - 1}{x}\right)$.

(B) Do as directed (Any two)   [08]

    (1) Evaluate: $\lim\limits_{x \to 0}\left(\dfrac{\cos ecx - \cot x}{x}\right)$

    (2) Evaluate: $\lim\limits_{n \to \infty}\left(\sqrt{n^2 + n + 1} - n\right)$

    (3) $f(x) = \dfrac{1 - x}{1 + x}$, then Prove That, $f(x) + f\left(\dfrac{1}{x}\right) = 0$ and $f(x) \times f(-x) = 1$.

Q.3 (A) Do as directed (Any two)   [06]

    (1) Differentiate $x^3 + 5x$ with respect to x using First Principle.

    (2) If $y = \log\left(x + \sqrt{x^2 + 1}\right)$, then find $\dfrac{dy}{dx}$.

(3) Find $\dfrac{dy}{dx}$ when (i) $y = e^x \sec x$ and (ii) $y = \dfrac{\log x}{x}$.

(B) Do as directed (Any two) [08]

(1) If $x + y = \sin(xy)$ then find $\dfrac{dy}{dx}$.

(2) Equation of motion of a particle is $S = t^3 - 5t^2 + 3t$. When particle comes to rest? Find acceleration at that time.

(3) Find Minimum and Maximum Value of the function $f(x) = x^3 - 4x^2 + 5x + 7$.

Q.4 (A) Do as directed (Any two) [06]

(1) Evaluate: $\displaystyle\int\left(\dfrac{x^3 + 5x^2 + 4x + 1}{x^2}\right)dx$

(2) Evaluate: $\displaystyle\int\left(\dfrac{\cos 2x}{\cos^2 x \sin^2 x}\right)dx$.

(3) Evaluate: $\displaystyle\int_0^1\left(\dfrac{2}{1+x^2}\right)dx$.

(B) Do as directed (Any two) [08]

(1) Evaluate: $\displaystyle\int\left(\dfrac{(1+x)e^x}{\cos^2(xe^x)}\right)dx$.

(2) Evaluate: $\displaystyle\int_0^{\frac{\pi}{2}}\left(\dfrac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\right)dx$.

(3) Show that the area enclosed between the parabola $y = x^2$ and lines $x = 2, x = 3$ And x-axis is 19/3 Sq. unit.

Q.5 (A) Do as directed (Any two) [06]

(1) Solve the diff.equation: $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

(2) If $\dfrac{(1+i)^2}{2-i} = x + iy$, Then find the value of x+y.

(3) Find complex conjugate and modulus of $z = \dfrac{1-i}{1+i}$.

(B) Do as directed (Any two) [08]

(1) Solve the diff.equation: $\dfrac{dy}{dx} = (x + y)^2$.

(2) Solve the diff.equation: $x\dfrac{dy}{dx} = y + x\cos^2\left(\dfrac{y}{x}\right)$.

(3) If $z = \dfrac{3}{1+\sqrt{2}i}$ then prove that, $z^2 - 2z + 3 = 0$. Find the value of $z^3 + z^2 - 3z + 10$ Using it.

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